DIFFUSION OF WATER IN MOIST MATERIALS

N. E. Gorobtsova

UDC 533.72:66.047

A method is given for calculating diffusion coefficients for a period of decreasing drying rate.

This paper is a continuation of an earlier one [2], which discussed the diffusion coefficient as a function of water content $\bar{u}$ for a period of constant drying rate in the absence of thermal transport:

$$a_m = \frac{NRR_V(\bar{u}_0 - \bar{u}_c)}{\Gamma(\bar{u}_0 - \bar{u})(\bar{u}_c - u_{mh})}, \tag{1}$$

with $\bar{u}_0 \geq \bar{u} \geq \bar{u}_c$. We put $u_c - u_e = B\bar{u}_0$ in (1), to give

$$a_m = \frac{(1-B)NRR_V}{B\Gamma(\bar{u}_0 - \bar{u})} \frac{1 - \dfrac{u_e}{\bar{u}_0(1-B)}}{1 - \dfrac{u_{mh} - u_e}{B\bar{u}_0}}, \tag{2}$$

or, with $u_{mh} = u_e/(1-B)$

$$a_m = \frac{1-B}{B\Gamma} \frac{NRR_V}{\bar{u}_0 - \bar{u}} = C\frac{NRR_V}{\Gamma\bar{u}_0}, \tag{3}$$

where $C = (1-B)/B\,(1-\bar{u}/\bar{u}_0)$.

Ermolenko's formula [4] for an unbounded plate can be used to examine a_m as a function of $\bar{u}$ in the period of decreasing rate:

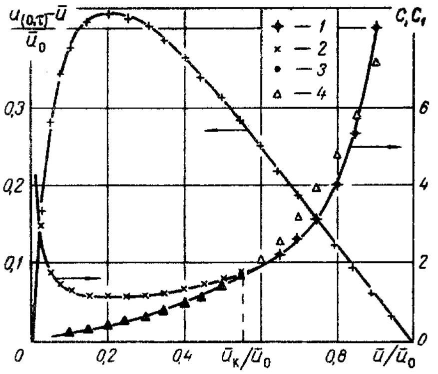

Fig. 1

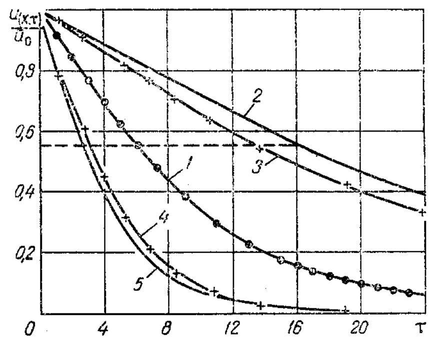

Fig. 2

Fig. 1. Graphs of $(u_{(0,\tau)} - \bar{u})/\bar{u}_0 = f(\bar{u}/\bar{u}_0)$ from (13), $C = f(\bar{u}/\bar{u}_0)$ [1 from (3), 2 from (23)], and $C_1 = f(\bar{u}/\bar{u}_0)$ [3 from (25), 4 from (26)].

Fig. 2. Drying curves (τ, h): 1) mean-integral curve, 2) and 3) at center as calculated from (5) and (27) respectively, 4) and 5) at surface, from (27) and (5).

Institute of Heat and Mass Transfer, Academy of Sciences of the Belorrusian SSR, Minsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 19, No. 1, pp. 27-33, July, 1970. Original article submitted December 29, 1969.

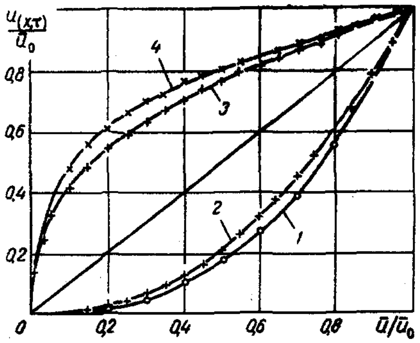

Fig. 3. Graphs of: 1) $u_{(1,\tau)}/\bar{u}_0 = f(\bar{u}/\bar{u}_0)$ from (5); 2) the same from (27); 3) $u_{(0,\tau)}/\bar{u}_0 = f(\bar{u}/\bar{u}_0)$. from (27); 4) the same from (5).

$$a_m = \frac{a\bar{u}}{a\tau}\left(R^2 - R_X^2\right)/6\left[u_{(X,\tau)} - \bar{u}\right]. \tag{4}$$

If a full representation of a_m as a function of $\bar{u}$ is to be obtained from (4), one needs not only the drying curve for the whole specimen but also the analogous curve for one of the layers [2], which is very tedious to obtain, mainly because there is difficulty in determining the local water content.

The problem becomes much simpler if we determine numerically the local water-content distribution from the mean-integral drying curve [3]:

$$u_{(X,\tau)} - u_e = \left[\bar{u}_0 - A_{(X)}N\tau_1 - u_e\right]\exp\left[-\frac{A_{(X)}N\tau_2}{\bar{u}_{rc} - u_e}\right], \tag{5}$$

where $A_{(X)}$ is a coefficient that is constant in time but that varies with the X coordinate:

$$A_{(X)} = \frac{\bar{u}_0 - u_{(X,\tau)}}{\bar{u}_0 - \bar{u}} = \Pi - 0.5\,\Gamma\,\frac{\bar{u}_c - u_{mh}}{\bar{u}_0 - \bar{u}_c}\left(\frac{\Pi}{\Gamma} - X^2\right), \tag{6}$$

and with $\bar{u}_c - u_e = B\bar{u}_0$ and $u_{mh} = u_e/(1-B)$

$$A_{(X)} = \Pi - 0.5\,\Gamma\,\frac{B}{1-B}\left(\frac{\Pi}{\Gamma} - X^2\right). \tag{7}$$

The time τ_1 is reckoned from the start of drying to τ_c, the time when the critical water content is reached, while τ_2 is reckoned from τ_c.

We have, as follows from (5), for an unbounded plate in the period of constant rate:

$$u_{(0,\tau)} - \bar{u} = \bar{u}_0 - \left(1 - 0.5\,\frac{B}{1-B}\right)\left(\bar{u}_0 - \bar{u}\right) - \bar{u} = \frac{0.5\,B}{1-B}\left(\bar{u}_0 - \bar{u}\right). \tag{8}$$

From (4) and (8) with $d\bar{u}/d\tau = N = \text{const}$ we have

$$a_m = \frac{NRR_V(1-B)}{2\Gamma\left(\bar{u}_0 - \bar{u}\right)0.5\,B} = \frac{(1-B)NRR_V}{B\Gamma\left(\bar{u}_0 - \bar{u}\right)}. \tag{9}$$

Equation (9) is analogous to (3).

Consider the period of decreasing rate. We see from (5) and (6) that τ_c for the plate center is

$$\frac{1}{A_{(X=0)}} = \frac{1-B}{1-1.5\,B} \tag{10}$$

times the τ_c for the mean over the plate, where $0.667 > B \geq 0$.

As $A_{(X=0)}$ is usually much less than one, the constant-rate period at the center ends at low mean-integral water contents, i.e., extends to almost the whole drying period.

Also, the total drying time equals the sum of the first period (water reduces from $\bar{u}_0$ to $\bar{u}_{rc}$) and the second period:

$$\tau_d = \frac{\bar{u}_0 - u_e}{N} - \frac{B\bar{u}_0}{N}\left(1 + 2.3 \lg \frac{\bar{u} - u_e}{B\bar{u}_0}\right), \tag{11}$$

or for $u_e = 0$

$$N\tau_d = \bar{u}_0\left(1 - B - 2.3 B \lg \frac{\bar{u}}{B\bar{u}_0}\right), \tag{12}$$

we get from (5)-(7) and (12) that the current water content at the center is

$$\frac{u_{(0,\tau)}}{\bar{u}_0} = 1 - \frac{1 - 1.5B}{(1 - B)\bar{u}_0} N\tau_d = 1.5\, B\left(1 + \frac{1.53 - 2.3B}{1 - B} \lg \frac{\bar{u}}{B\bar{u}_0}\right). \tag{13}$$

One can use (13) to find $u_{(0,\tau)}/\bar{u}_0$ in the second period, and so one can find a_m from (4)

$$a_m = \frac{0.5 \dfrac{d\bar{u}}{d\tau} RR_V}{\Gamma\bar{u}_0\left[1.5B\left(1 + \dfrac{1.53 - 2.3B}{1 - B} \lg \dfrac{\bar{u}}{B\bar{u}_0}\right) - \dfrac{\bar{u}}{\bar{u}_0}\right]} \tag{14}$$

or for $d\bar{u}/d\tau = N\bar{u}/B\bar{u}_0$

$$a_m = \frac{0.5NRR_V}{\Gamma B\bar{u}_0\left[1.5B \dfrac{\bar{u}_0}{\bar{u}} + B\dfrac{\bar{u}_0}{\bar{u}}\left(\dfrac{2.3 - 3.45B}{1 - B}\right) \lg \dfrac{\bar{u}}{B\bar{u}_0} - 1\right]}. \tag{15}$$

Equation (15) for $0 \le B > 0.667$ is represented closely by

$$a_m = a_{mc} \exp\left[4\left(\frac{\bar{u}}{\bar{u}_0} - B\right)\right], \tag{16}$$

and since from (3)

$$a_{mc} = \frac{(1 - B) NRR_V}{B\Gamma(\bar{u}_0 - B\bar{u}_0)} = \frac{NRR_V}{B\Gamma\bar{u}_0}, \tag{17}$$

we have

$$a_m = \frac{NRR_V}{B\Gamma\bar{u}_0} \exp\left[4\left(\frac{\bar{u}}{\bar{u}_0} - B\right)\right] = C_1 \frac{NRR_V}{\Gamma\bar{u}_0}, \tag{18}$$

where

$$C_1 = B^{-1} \exp\left[4\left(\frac{\bar{u}}{\bar{u}_0} - B\right)\right]. \tag{19}$$

Equation (18) can be given the following convenient form:

$$a_m = a_0 \exp\left(4\frac{\bar{u}}{\bar{u}_0}\right), \tag{20}$$

where

$$a_0 = \frac{NRR_V}{B\Gamma\bar{u}_0} \exp(-4B).$$

We examine $a_m = f(\bar{u}/\bar{u}_0)$ via the common case $\bar{u}_{rc} - u_e = \bar{u}_c - u_e = B\bar{u}_0 = \bar{u}_0/1.8$, i.e., $B = 0.556$ [1], whereupon (14) with $B = 0.556$ gives

$$a_m = \frac{0.523 \frac{d\bar{u}}{d\tau} RR_V}{\Gamma \bar{u}_0 \left(1 + 0.5 \lg \frac{\bar{u}}{\bar{u}_0} - 1.05 \frac{\bar{u}}{\bar{u}_0} \right)}, \tag{21}$$

or

$$a_m = \frac{C \frac{d\bar{u}}{d\tau} RR_V,}{\Gamma \bar{u}_0}, \tag{22}$$

where

$$C = \frac{0.523}{1 + 0.5 \lg \frac{\bar{u}}{\bar{u}_0} - 1.05 \frac{\bar{u}}{\bar{u}_0}}. \tag{23}$$

Figure 1 shows $C = f(\bar{u}/\bar{u}_0)$ for the entire period, and derived from (3) for the first period. Equations (3) and (22) differ only in the drying rate, the first period having N = constant and the second having $d\bar{u}/d\tau$ variable.

The $C = f(\bar{u}/\bar{u}_0)$ curve shows that C falls rapidly in the first period, while in the second it falls less rapidly, then becomes almost constant, and finally rises again. If the drying rate is constant, the $a_m = f(\bar{u}/\bar{u}_0)$ curve is similar to the $C = f(\bar{u}/\bar{u}_0)$ curve because all the quantities in (3) and (22) are constant, apart from C. In other cases the shape of $a_m = f(\bar{u}/\bar{u}_0)$ is determined by C and $d\bar{u}/d\tau$ or C_1.

For instance, for $d\bar{u}/d\tau = 1.8N\bar{u}/\bar{u}_0$, i.e., B = 0.556

$$a_m = \frac{0.942 NRR_V}{\bar{u}_0 \Gamma \left(\frac{\bar{u}_0}{\bar{u}} + 0.5 \frac{\bar{u}_0}{\bar{u}} \lg \frac{\bar{u}}{\bar{u}_0} - 1.05 \right)} = \frac{C_1 NRR_V}{\Gamma \bar{u}_0}, \tag{24}$$

where

$$C_1 = \frac{0.942}{\frac{\bar{u}_0}{\bar{u}} + 0.5 \frac{\bar{u}_0}{\bar{u}} \lg \frac{\bar{u}}{\bar{u}_0} - 1.05} = \frac{C\bar{u}}{B\bar{u}_0}. \tag{25}$$

Figure 1 also shows $C_1 = f(\bar{u}/\bar{u}_0)$, which is closely fitted by (19) with B = 0.556:

$$C_1 = 0.195 \exp\left(4 \frac{\bar{u}}{\bar{u}_0} \right). \tag{26}$$

Equation (26) can be extended to the entire drying period.

The above method can be simplified considerably in order to calculate transient water contents and diffusion coefficients. Equation (5) may be represented closely as

$$\frac{u_{(X,\tau)} - u_e}{\bar{u}_0 - u_e} = \left(\frac{\bar{u} - u_e}{\bar{u}_0 - u_e} \right)^{A(X)} \tag{27}$$

and so one needs only the mean-integral drying curve to determine transient water distributions.

Figure 2 shows $u_{(0,\tau)}/\bar{u}_0 = f(\bar{u}/\bar{u}_0)$ and $u_{(1,\tau)}/\bar{u}_0 = f(\bar{u}/\bar{u}_0)$ derived from (5) and (27) with B = 0.556 and u_e. The maximum discrepancy between the values calculated from (5) and (27) is 20% (see Fig. 3).

One can easily trace the change in transient water contents in the thickness from $u_{(X,\tau)}/\bar{u}_0 = f(\bar{u}/\bar{u}_0)$, e.g., in Fig. 3 are shown the limits of $u_{(X,\tau)}/\bar{u}_0 = f(\bar{u}/\bar{u}_0)$ i.e., curves for $u_{(0,\tau)}/\bar{u}_0 = f(u/\bar{u}_0)$ and $u_{(1,\tau)}/\bar{u}_0 = f(\bar{u}/\bar{u}_0)$ for an unbounded plate for B = 0.556. The other $u_{(X,\tau)}/\bar{u}_0 = f(\bar{u}/\bar{u}_0)$ curves are similar, and for X < 0.577 they lie in the upper region bounded by the curve $u_{(0,\tau)}/\bar{u}_0 = f(\bar{u}/\bar{u}_0)$ and the line $u_{(0.577;\tau)}/\bar{u}_0 = f(\bar{u}/\bar{u}_0)$, while for X > 0.577 they lie in the region bounded by the same line and the curve $u_{(1,\tau)}/\bar{u}_0) = f(\bar{u}/\bar{u}_0)$.

Equation (27) greatly simplifies the determination of diffusion coefficients from (4), e.g., for an unbounded plate:

$$a_m = \frac{0.5 \frac{d\bar{u}}{d\tau} RR_V}{\Gamma\left(\bar{u}_0^{\frac{0.5B}{1-B}} \bar{u}^{\frac{1-1.5B}{1-B}} - \bar{u}\right)}, \tag{28}$$

and if $d\bar{u}/d\tau = N\bar{u}/B\bar{u}_0$, then

$$a_m = \frac{0.5NRR_V}{\Gamma B \bar{u}_0 \left[\left(\frac{\bar{u}}{\bar{u}_0}\right)^{-\frac{0.5B}{1-B}} - 1\right]}. \tag{29}$$

NOTATION

$\bar{u}_0$, $\bar{u}$, u_c, u_{rc}, $u(X, \tau)$, $u(0, \tau)$, $u(1, \tau)$, u_{mh}, u_e	initial, mean-integral, critical, and reduced critical, local moisture contents, moisture content at centre of material, moisture content at surface of material, maximum hygroscopic, and equilibrium moisture contents, respectively;
a_m, a_{mc}, a_0	moisture diffusivity: mean-integral, at critical moisture content, and of absolutely dry material, respectively;
$d\bar{u}/d\tau$, $du_X/d\tau$, N	mean-integral, layer, and local drying rates, respectively in the period of constant drying rate;
τ_1, τ_2, τ_c	time at constant drying rate, at decreasing rate, and at u_{rc}, respectively;
τ_d	total drying time;
A(X), B, C, C_1	numerical coefficients;
Γ, Π	constant numerical coefficients: for an infinite plate, Π = 1, Γ = 3, for an infinite cylinder, Π = 2, Γ = 4; for a sphere, Π = 3, Γ = 5;
R	characteristic dimension of body;
R_V	hydraulic radius;
R_X	thickness of sample layer;
X	dimensionless coordinate: for infinite plate, X = x/R, where x is the running coordinate; for a cylinder or sphere, X = r/R, where r is radius.

LITERATURE CITED

1. A. V. Lykov, Theory of Drying [in Russian], Énergiya, Moscow (1968).
2. N. E. Gorobtsova, Inzh.-Fiz. Zh., 15, No. 2 (1968).
3. N. E. Gorobtsova, Inzh.-Fiz. Zh., 17, No. 2 (1969).
4. V. D. Ermolenko, Inzh.-Fiz., No. 10 (1962).